

Optimal Linear Transmit Beam Forming Techniques for Multi-User MIMO

Patteti Krishna¹, Soma Umamaheshwar², Tipparti Anil Kumar³,

⁴Kalithkar Kishan Rao and ⁵Kunupalli Srinivas Rao⁵

¹ECE Department, SVS Group of Institutions, Warangal, Telangana, India – 506 015

²ECE Department, Varadhareddy Engineering College, Warangal, Telangana, India – 506 371

³ECE Department, SR Engineering College, Warangal, Telangana, India – 506 371

⁴Vaagdevi College of Engineering, Warangal, Telangana, India – 506 005

⁵TRR College of Engineering, Hyderabad, Telangana, India – 502 319

¹kpatteti@gmail.com, ²umamaheshwarsoma@rediffmail.com, ³tvakumar2000@yahoo.co.in

⁴prof_kkr@rediffmail.com and ⁵principaltrr@gmail.com

ABSTRACT

Adaptive transmit beam forming is key to increased spectral and energy efficiency in next-generation wireless networks. In light of the difficulty to compute the optimal multiuser transmit beam forming there is a plethora of heuristic schemes. Transmit beam forming is a versatile technique for signal transmission from an array of N antennas to one or multiple users. In wireless communications, the goal is to increase the signal power at the intended user and reduce interference to non-intended users. A high signal power is achieved by transmitting the same data signal from all antennas. Since transmit beam forming focuses the signal energy at certain places, less energy arrives to other places. This allows for so-called space-division multiple accesses (SDMA), where K spatially separated users are served simultaneously. One beam forming vector is assigned to each user and can be matched to its channel. Unfortunately, the finite number of transmit antennas only provides a limited amount of spatial directivity, which means that there are energy leakages between the users which act as interference. To design a beam forming vector that maximizes the signal power at the intended user, it is difficult to strike a perfect balance between maximizing the signal power and minimizing the interference leakage.

Keywords: SDMA; Transmit Beam forming; SNIR and Power minimization.

I. INTRODUCTION

Multi-input multi-output (MIMO) communications systems have attracted high spectral efficiency. In point-to-point multiple antenna systems it is well known that the capacity increases linearly with the minimum of the number of transmit/receive antennas, irrespective of the availability of channel state information (CSI) at the base station.

The multi-user MIMO downlink refers to where multi-antenna transmitter simultaneously communicates with several co-channel users. Only

recently has the multi-user MIMO downlink been addressed, beginning with information-theoretic capacity results [1–5], and followed by practical implementations, including those based on linear techniques [6, 7] and non-linear pre coding [8–11].

The problem of meeting quality of services (QoS) constraints with minimum transmit power is often referred to as the downlink power control or interference-balancing problem. As with sum capacity maximization, channel knowledge at the

transmitter is crucial to finding a solution. Channel state information is most often obtained by means of uplink training data, as in a time-division duplex system, or via feedback from the users, as in the frequency-division duplex case. Each approach has its advantages and disadvantages in terms of throughput penalty and latency.

CSI can be in the form of deterministic channel estimates, or it can be described in probabilistic terms (e.g., channel mean and covariance). While we will focus on the deterministic case in this chapter, statistical CSI may be directly applied in most cases. For an excellent and comprehensive treatment of the issues involved with different types of CSI.

In line-of-sight (LOS) between the transmitter and receiver, beam forming can be seen as forming a signal beam toward the receiver. Figure.1 Beam forming can also be applied in non-LOS scenarios, if the multipath channel is known, by making the multipath components add coherently or destructively. Since transmit beam forming focuses the signal energy at certain places, less energy arrives to other places. This allows for so-called space-division multiple access (SDMA), where K spatially separated users are served simultaneously. One beam forming vector is assigned to each user and can be matched to its channel. Unfortunately, the finite number of transmit antennas only provides a limited amount of spatial directivity, which means that there are energy leakages between the users which act as interference. While it is fairly easy to design a beam forming vector that maximizes the signal power at the intended user, it is difficult to strike a perfect balance between maximizing the signal power and minimizing the interference leakage. In fact, the optimization of multiuser transmit beam forming is generally a nondeterministic polynomial-time (NP) hard problem.

The rest of this paper is organized as follows. The next section we introduce the power minimization with signal to interference ratio. In the section III, we describe the results analysis and finally conclusions are drawn in section IV.

II. POWER MINIMIZATION WITH SINR

We consider a downlink channel where a base station (BS) equipped with N antennas

communicates with K single-antenna users using SDMA. The data signal to user k is denoted $p_k \in \mathbb{R}$ and is normalized to unit power, while the vector $h_k \in \mathbb{C}^{N \times 1}$ describes the corresponding channel. The K different data signals are separated spatially using the linear beam forming vectors w_1, w_2, \dots, w_K where w_k is associated with user k .

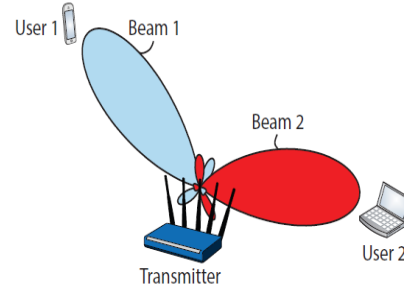


Fig. 1. Visualization of transmit beam forming in an LOS scenario.

The received signal at k user as

$$y_k = h_k^H \left(\sum_{i=1}^K w_i p_i \right) + z_k \quad (1)$$

Where z_k is additive receiver noise with zero mean and variance σ^2 consequently, the signal-to-noise and- interference ratio (SINR) at user k is

$$\text{SINR}_k = \frac{|h_k^H w_k|^2}{\sum_{i \neq k} |h_k^H w_i|^2 + \sigma^2} = \frac{\frac{1}{\sigma^2} |h_k^H w_k|^2}{\sum_{i \neq k} \frac{1}{\sigma^2} |h_k^H w_i|^2 + 1} \quad (2)$$

The transmit beam forming can be optimized to maximize some performance utility metric, which is generally a function of the SINRs. We first solve the relatively simple power minimization problem

$$\min_{w_1, \dots, w_K} \sum_{k=1}^K \|w_k\|^2 \quad (3)$$

$$\text{SINR}_k \geq \gamma_k \quad (4)$$

The parameters $\gamma_1, \dots, \gamma_K$ are the SINRs that each user shall achieve at the optimum of (4), using as little transmit power as possible. The γ -parameters can, for example, describe the SINRs required for achieving certain data rates. The absolute values in the SINRs in (2) make w_k and $e^{j\theta_k}$ completely equivalent for any common phase rotation $\theta_k \in [0, 2\pi)$. Without loss of optimality, we exploit this phase ambiguity to rotate the phase such that the inner

product $h_k^H w_k$ is real-valued and positive. This implies that $\sqrt{|h_k^H w_k|^2} = h_k^H w_k \geq 0$. By letting \square denoting the real part, the constraint $SINR_k \geq \gamma_k$ can be rewritten as

$$\frac{1}{\gamma \sigma^2} |h_k^H w_k|^2 \geq \sum_{i \neq k} \frac{1}{\sigma^2} |h_k^H w_i|^2 + 1 \Leftrightarrow \frac{1}{\sqrt{\gamma_k \sigma^2}} \Re h_k^H w_k \geq \sqrt{\sum_{i \neq k} \frac{1}{\sigma^2} |h_k^H w_i|^2 + 1} \quad (5)$$

The reformulated SINR constraint in (5) is a second-order cone constraint, which is a convex type of constraint [10]–[12], and it is easy to show that Slater's constraint qualification is fulfilled [13]. Hence, optimization theory provides many important properties for the reformulated convex problem; in particular, strong duality and that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for the optimal solution. The optimal beam forming vectors

$$w_k^* = \sqrt{p} \frac{\left(I_N + \sum_{i=1}^K \frac{\lambda_i}{\sigma^2} h_i h_i^H \right)^{-1} h_k}{\left\| \left(I_N + \sum_{i=1}^K \frac{\lambda_i}{\sigma^2} h_i h_i^H \right)^{-1} h_k \right\|} \quad (6)$$

For $k=1, 2, \dots, K$

$$= \sqrt{p_k} \cdot \hat{w}_k^* = \text{beam forming direction} \quad (7)$$

Where \hat{w}_k^* denotes the beam forming power and \hat{w}_k^* denotes the unit-norm beam forming direction for user k . The K unknown beam forming powers are computed by noting that the SINR constraints (4) hold with equality at the optimal solution.

III. RESULTS ANALYSIS

In this section we provides the properties of Maximum Ratio Transmission (MRT), Zero Forcing Beam Forming (ZFBF), and transmit MMSE beam forming are illustrated by simulation in Figure 2. We consider $K = 4$ users with the sum rate as utility function:

$$f \text{ SINR}_1, \dots, \text{SINR}_4 = \sum_{k=1}^4 \log_2 (1 + \text{SINR}_k) \quad (8)$$

Figure 2 shows the simulation results for (a) $N = 4$ and (b) $N = 12$ transmit antennas. In the former case, we observe that MRT is near-optimal at low SNRs, while ZFBF is asymptotically optimal at high-SNRs. Transmit MMSE beam forming is a more versatile scheme that combines the respective asymptotic properties of MRT and ZFBF with good performance at intermediate SNRs. However, there is still a significant gap to the optimal solution, which is only bridged by fine-tuning the $K = 4$ parameters $\lambda_1, \dots, \lambda_4$ (with an exponential complexity in K). In the case of $N = 12$, there are many more antennas than users, which makes the need for fine-tuning much smaller; transmit MMSE beam forming is near optimal in the entire SNR range.

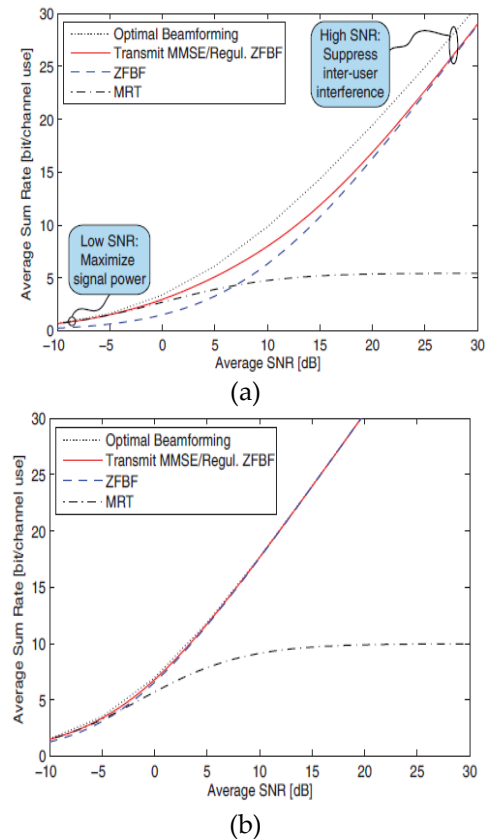


Fig. 2. Average sum rate for $K = 4$ users as a function of the average SNR.

Heuristic beam forming can perform closely to the optimal beam forming, particularly when there are many more antennas than users. Transmit MMSE/regularized ZFBF always performs better, or equally well, as MRT and ZFBF.

IV. CONCLUSIONS

The optimal beam forming maximizes the received signal powers at low SNRs, minimizes the interference leakage at high SNRs, and balances between these conflicting goals at intermediate SNRs. In this paper describes the optimal beam forming structure can be extended to practical multi-cell scenarios. Alternative beam forming parameterizations based on local channel state information (CSI) or transceiver hardware impairments can be found. Some open problems in this field are the robustness to imperfect CSI, multi-stream beam forming to multi-antenna users, multi-casting where each signal is intended for a group of users

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